LUDOVICI A RIPA ... DISSERTATIO METEOROLOGICA. CUI ACCEDIT SOLUTIO & CONSTRUCTIO...

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SOLUTIO, *ET* CONSTRUCTIO.

DUORUM PROBLEMATUM GEOMETRICORUM.

PROBLEMA I.

INvenire minimam lineam quæ duci poteſt per punctum datum A intra angulum datum F D G. Fig. I.

ANALYSIS.

Sit C B minima quęſita, Ducatur Q A normalis F D, A P normalis B D, C K normalis F P. Igitur cognitę erunt A F, AQ, QD. Ponatur QF $= a$. QA $= c$. QD $= b$. QC $= x$, erit FA $= \sqrt{cc+aa}$, CA $= \sqrt{xx+cc}$, FP $= \frac{\overline{a+b}.a}{\sqrt{cc+aa}}$ ob analogiam FA: F Q:: FD: FP, igitur AP $= \frac{ab-cc}{\sqrt{cc+aa}}$. FK $= \frac{d.\overline{a-x}}{\sqrt{cc+aa}}$ ob analogiam A F: FQ:: FC: FK, igitur AK $= \frac{cc+ax}{\sqrt{aa+cc}}$. Sed cum

 ſit

ſit AK : CA :: AP : AB $= \frac{\overline{ab - cc} \cdot \sqrt{xx+cc}}{cc+ax}$ erit BC $= \sqrt{cc+xx} + \frac{\overline{ab - cc} \cdot \sqrt{cc+xx}}{cc+ax}$ Preterea per conditionem *Minimi* habebimus

$$\frac{xdx + \overline{ab - cc} \cdot \overline{cc+ax} \cdot xdx - adx \cdot \overline{cc+xx}}{\sqrt{cc+xx} \quad \overline{cc+xa}^2 \cdot \sqrt{cc+xx}} = 0$$

Æquatione reducta inveniemus $\overline{b+x} \cdot x = \frac{\overline{ba - cc} \cdot \sqrt{cc+xx}}{cc+ax} \cdot \sqrt{cc+xx}$, nempe DC . QC $=$ AB . GA.

CONSTRUCTIO.

Ducatur DE normalis CB, & deſcriptus intelligatur circulus AQDPE comprehendens angulum datum cum ſit CE . CA $=$ CQ . CD, erit CE . CA $=$ AB . CA, ergo AB $=$ CE, ergo CA $=$ EB.

Nunc exponam praxim ingenioſi Architecti, quam adhibuit pro ſolutione hujus Problematis inſervientis ad determinandam maximam longitudinem, quæ volvi poteſt circa punctum, tanquam obicem, in dato angulo. *Ex dato puncto A duxit lineam AD intra angulum CDB, & circulum deſcripſit Diametro AD, deinde circa punctum A revolvere*

cœpit

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cœpit regulam C B, *& motum hunc tunc ceſſare fecit cum invenit circino* $AC = EB$. *His abſolutis adfirmavit lineam* C B *eſſe maximam longitudinem, quæ in angulo dato volvi poteſt circa punctum datum*. Animadverto puncta A & E eſſe in hyperbola tranſeunte per punctum A. Intra aſymptotos F D, DG hyperbola S A E deſcribatur, quę tranſeat per punctum A. Deſcripta hyperbola ſcindet circulum in E, dico lineam C A E B eſſe quæſitam.

Idem Problema aliter.

Esto $QA = c$. $QD = b$. $AP = n$. $PD = m$. $CQ = x$. $BP = z$. $AC = \sqrt{cc+xx}$. $AB = \sqrt{nn+zz}$. Jam cum ſit $bx+xx = xx+cc+AE.\sqrt{cc+xx}$ erit $AE = \frac{bx-cc}{\sqrt{cc+xx}}$; & cum ſit $mz+zz = zz+nn-AE.\sqrt{zz+nn}$, erit $AE = \frac{nn-mz}{\sqrt{zz+nn}} = \frac{bx-cc}{\sqrt{cc+xx}}$. Sed $AB = \sqrt{cc+xx} + \sqrt{nn+zz}$ igitur per conditionem *Minimi* habebimus $\frac{xdx}{\sqrt{cc+xx}} + \frac{zdz}{\sqrt{nn+zz}} = 0$. Poſt debitas reductiones, & ſubſtitutiones oritur ſequens Conſtructio.

CON-

CONSTRUCTIO.

Fig. III. Lineę PD, QD bifariam dividantur in puctis F & G, a quibus ducantur FE, GE parallelę lineis PD, QD. Centro facto in puncto concurſus E ſuper axim GE deſcribatur hyperbola, cujus coordinatę angulum D efficiant, ejuſque Diameter ſit $\sqrt{\overline{QD}^2 - \overline{PD}^2}$ ſi vero QD minor eſſet PD, conſtruatur hyperbola ſuper axem FE iiſdem conditionibus, mutatis tamen ſignis in valore Diametri. Præterea ſuper axem FE, centro E, diametro $\sqrt{4\overline{AQ}^2 - 4\overline{AP}^2 + 3\overline{QD}^2 - 3\overline{PD}^2}$ alia conſtruatur hyperbola, cujus quoque coordinatę, faciant angulum D, ſi autem diametri valor eſſet imaginarius, mutatis ſignis ejuſdem Diametri ſuper axem EG deſcribatur hyperbola. Hę hyperbolę ſe invicem ſecabunt in K. Deinde a puncto K agantur KB, KC parallelę CD, BD. Dico lineam CB eſſe quęſitam. Multas determinationes hujus Problematis brevitatis cauſa omittimus.

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PROBLEMA II.

Dato puncto A in dato angulo FDG invenire minimam tangentem circuli deſcripti a radio AL centro A intra datum angulum Fig. IV.

ESto NK tangens queſita. $AQ = c$. $QF = a$. $QD = b$. $AL = b$. $QM = z$. $AM = \sqrt{zz+cc}$ $QC = \frac{cc}{z}$. $AP = \frac{ab - cc}{\sqrt{aa+cc}}$

$CA = \frac{c\sqrt{zz+cc}}{z}$. $AB = \frac{ab - cc \,.\, \sqrt{zz+cc}}{cz + ac}$.

Hęc omnia oriuntur ex analyſi pręcedentis Problematis. $QM : QA :: ML : NL$ ergo $NL = \frac{c\sqrt{zz+cc} - cb}{z}$. $AT : TC ::$ $AP : PB$, ergo $PB = \frac{ab - cc \,.\, az - cc}{cz + ac \,.\, \sqrt{aa+cc}}$ PB:

$AP : AB : AR$ igitur $AR = \frac{ab - cc \,.\, \sqrt{zz+cc}}{az - cc}$

& $LR = \frac{ab - cc\sqrt{zz+cc}}{az - cc} - b$. $RL : LK ::$ $RA : AB$ ergo $LK = \frac{ab - cc\sqrt{zz+cc} - abz + ccb}{cz + ac}$.

Igitur

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Igitur NK $= \dfrac{\overline{acc+abz}\cdot\sqrt{zz+cc}-abz^2-accb}{czz+acz}$.

Per conditionem *Minimi* erit

$$\frac{\overline{zz+az}\cdot\overline{abdz\sqrt{zz+cc}+acc+abz}\cdot zdz-2abzdz}{c\cdot\overline{zz+az}^{2}}$$

$$\frac{-\overline{acc-abz-zz+cc}+ab\cdot\overline{zz+cc}^{\frac{3}{2}}\cdot\overline{2zdz+adz}}{c\cdot\overline{zz+az}^{2}}=0.$$

Ex hujus equationis reductione prodibit ſequens equatio $2bccz+abcc-abzz\cdot\sqrt{zz+cc}=$ $ac^4+c^2z^3-abz^3+bccz^2+2c^4z$, *ex qua oritur* $\dfrac{ab-cc\sqrt{cc+zz}-b\cdot\sqrt{zz+cc}-b}{az-cc}=$

$$\frac{\overline{bz+cc-b\sqrt{cc+zz}}^{2}}{\sqrt{zz+cc}^{2}}$$ hoc eſt RL . ML $=$

$\overline{ED}^2$ hoc eſt RL: ED:: ED: M L; Sed LR: ED:: LK: EK, ergo ED: ML:: LK: EK. Sed ED: ML:: NE: N L. ergo LK: EK:: NE: LN. Igitur LK— EK: EK:: NE - NL: NL, hinc patet eſſe NL$=$EK.

Apponam aliam praxim a laudato Architecto excogitatam, ut

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ut Problemati huic ſatisfaceret. quod tali pacto ei propoſitum fuit, *dato angulo, & dato puncto intra eundem angulum, & data latitudine lecticę quæritur maxima longitudo illius rectanguli conſtituentis baſim ejuſdem lecticæ, quæ volvi poteſt circa datum punctum, tanquam obicem, in dato angulo*. Fig. II. *Ex puncto A tanquam centro circulum deſcripſit radio æquali latitudini lecticæ, deinde duas normas ſupra regulam aptavit, quas movere fecit tali lege, ut dum latus unum primæ normæ tranſit per punctum A, & latus unum ſecundæ tranſit per verticem anguli D, reliqua duo latera normarum adhæreant regulæ X Z, & alterum latus normæ A E tangat circumferentiam circuli deſcripti; & motum hunc tunc ceſſare fecit, cum invenit circini ope regulæ partem* E C = F P. *His peractis confidenter aſſeruit* C B *eſſe maximam longitudinem lecticæ, quæ volvi*

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poteſt in angulo CDB circa punctum A. Quę mechanica conſtructio convenit cum iis, quę Analyſeos pręſidio deteximus.

Idem aliter.

SIt $CQ = x$. $PB = z$. $QA = c$. $QD = b$. $PD = m$. $AP = n$. $CD = b+x$. $BD = m+z$, & $bx+xx = AS.\sqrt{xx+cc} + xx+cc$, ergo $AS = \frac{bx-cc}{\sqrt{xx+cc}}$. Pariter $mz+zz = zz+nn - AS.\sqrt{zz+nn}$ ergo $AS = \frac{nn-mz}{\sqrt{zz+nn}} = \frac{bx-cc}{\sqrt{xx+cc}}$.

Sed $AP : PB :: KV : VB = \frac{bz}{n}$, & $QA : QC :: NG : CG = \frac{bx}{c}$, igitur $NK = \sqrt{cc+xx} - \frac{bx}{c} + \sqrt{zz+nn} - \frac{bz}{n} =$. Per conditionem *Minimi* erit $\frac{xdx}{\sqrt{cc+xx}} - \frac{bdx}{c} + \frac{zdz}{\sqrt{zz+nn}} - \frac{bdz}{n} = 0$.

Peractis debitis ſubſtitutionibus, & reductionibus inveniemus duas equationes $\frac{bx-cc}{\sqrt{cc+xx}} = \frac{nn-mz}{\sqrt{nn+zz}}$, & $\sqrt{xx+cc} - \frac{bx}{c} = \frac{zz+mz}{\sqrt{nn+zz}} - \frac{bz}{n}$

Quæ

Quæ curvę conſtructę ex puncto X intra angulum OXH ductis PX, QX parallelis QD, PD ſe invicem ſecabunt. Ex puncto ſectionis ducantur parallelę lateribus anguli dati, quæ determinabunt puncta B, C, & rectam CB, agatur NK parallela CB in diſtantia AL, dico NK eſſe lineam quæſitam.

CONSTRUCTIO *Curvę* $\frac{bx - cc}{\sqrt{cc+xx}} = \frac{nn - mz}{\sqrt{nn+zz}}$.

FIg. V. Sit angulus BAC = dato, aut ejus complemento ad duos rectos, & centro A conſtruatur hyperbola $zz = qq - nn$, cujus ordinatę ſint parallelę FA. Sit CA $= \frac{n}{b} =$ CH, & ducatur AH. Sit AC: CD:: $b:m$. Prętereа ſit CS $= \frac{cc}{b}$, & centro S ſuper axem SQ parallelum AB deſcribatur hyperbola $xx = tt - cc$, cujus ordinatę ſint parallelę CA. Ducatur ad libitum PE parallela CA. Per O ducatur OF parallela AB, a puncto L ducatur LM parallela FA; a puncto M agatur MN parallela AB; a puncto N ducatur per C linea NCT; a puncto T

 linea

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linea TR parallela CA. Dico OE $= z$. TR $= x$, & ducta TX parallela RS, & GX parallela CA, dico punctum X eſſe in Curva $\frac{bz - cc}{\sqrt{cc + zz}} = \frac{nn - mx}{\sqrt{nn + xx}}$.

DEMONSTRATIO.

CUm ſit OE $=$ FA $= z$ ex *hypoth.*, & ex *conſtruc.* AC: CD :: $b : m$ erit FL $= \frac{mz}{b} =$ NE per *naturam trianguli* AFG; igitur PN $= \frac{nn - mz}{b}$ cum poſitum fuerit CA $= \frac{nn}{b}$. Sed per *hyperbolam* BO eſt AE $=$ CP $= \sqrt{zz + nn}$, ergo CP: PN :: $\sqrt{zz + nn} : \frac{nn - mz}{b}$. Sed CP: PN :: CV: TV, & CV $= \sqrt{xx + cc}$ per *hyperbolam* QT, & TV $= x - \frac{cc}{b}$ cum ſit CS $= \frac{cc}{b}$. Ergo $\sqrt{zz + nn} : \frac{nn - mz}{b} :: \sqrt{xx + cc} : x - \frac{cc}{b}$. Tandem $\frac{xb - cc}{\sqrt{xx + cc}} = \frac{nn - mz}{\sqrt{zz + nn}}$. Q. E. D.

CON-

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CONSTRUCTIO *Curvæ*

$$\sqrt{xx+cc} - \frac{bx}{c} = \frac{zz+mz}{\sqrt{nn+zz}} - \frac{bz}{n}.$$

FIg. VI. Sit angulus BCK ęqualis dato, & centro C ſuper axem CF conſtruatur hyperbola $zz = qq - nn$, cujus ordinatę ſint parallelę CK. Accipiatur CA $= m$, ducaturque AG parallela CF, & ſit CA: BC :: $b : c$. Ducatur CQ, ita ut ut ſemper ſit CK = KQ, & agatur CL, ita ut ſit CK: KL :: $n : b$. Superaxem CP centro C deſcribatur hyperbola $xx = tt - cc$, cujus ordinatę ſint parallelę BC. Ad libitum ducatur NF, deinde NK parallela CF, tandem QD parallela NF. Conjungantur puncta A & N, & a puncto C ducatur CM parallela AN, producaturque DQ. Ex puncto L agatur LH parallela CQ. Conjungantur puncta K & M, ducaturque HE parallela KM; a puncto E linea EI parallela CF, a puncto I liena IO parallela BA, deinde linea OP parallela CF, dico OP $= x$ NF $= z$, & punctum X eſſe in Curva.

DEMONSTRATIO.

EX *hyppoth.* NF $= z$, ergo KQ = KC = CD = z per *naturam Trianguli* CKQ, & *parallelarum*, & per *hyperbolam*

AG

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$AG = CF = \sqrt{zz+nn}$. Sed AG : GN :: CD : DM, ergo $DM = \frac{zz+mz}{\sqrt{zz+nn}}$ cum ex *conſtruc.* ſit $FG = CA = m$. Sed CK : KL :: $n : b$, igitur $KL = \frac{bz}{n} = KH = ME$ per *naturam Trianguli* CQK, & *parallelarum.* $ED = \frac{zz+mz}{\sqrt{zz+nn}} - \frac{bz}{n} = CI$. Denominato OP $= x$, & per *hyperbolam* determinato $CP = \sqrt{xx+cc}$ erit $PI = \sqrt{xx+cc} - \frac{zz - mz}{\sqrt{zz+nn}} + \frac{bz}{n}$ At CA : BC :: IP : PO :: $b : c$, ergo $\frac{bx}{c} = \sqrt{xx+cc} - \frac{zz - mz}{\sqrt{zz+nn}} + \frac{bz}{n}$. Q. E. D.

Conſulto ommittimus determinationes hujus problematis, animadvertimus tamen traditas conſtructiones Curvarum facile abſolvi poſſe ſubſidio cujuſdam machinę unico, & continuo motu deſcribentis curvam: in conſtructione hujus machinę nil aliud ſupponi debet, pręter deſcriptiones vulgares & notas hyperbolarum. Artificium hujus inſtrumen-

menti, adhibita mediocri ſolertia & induſtria juxta varias circumſtantias Problematum, transſduci poteſt, ad conſtruendam quamlibet aliam curvam; & hoc pacto Geometria longius provehi poſſet, methodum conſtructionum reddendo facilem, uſui promptam, ac paratam. Hoc nunc indicaſſe ſatis ſit.

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Solutiones, & Conſtructiones duorum Problematum Geometricorum.

Venetiis Anno MDCCXXXIII.

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